(A) NUMBER SYSTEM (ANSWERS) ⟨12⟩ [1]

1. (b) Let the cost of each sharpner and each eraser be ₹ s and ₹ e respectively.

$5s + 6e = 28 \longrightarrow$ (i)

$6s + 5e = 27 \longrightarrow$ (ii)

Adding both equations; $11(s+e) = 55$

$\Rightarrow s + e = 5 \longrightarrow$ (iii)

Now; eqⁿ(ii) − eqⁿ(i) ⟶ We get; $s - e = -1 \rightarrow$ (iv)

Now; adding eqⁿ(iii) & eqⁿ(iv); We get → $s = 2$

then; $s + e = 5 \Rightarrow 2 + e = 5 \Rightarrow e = 3$

∴ Required answer is → $\boxed{2, 3}$

2. (b) Two equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, Where x and y are Variables and a_1, b_1, a_2, b_2, c_1 and c_2 are all constants Which Will have infinite solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ According to the question:

Here; $a_1 = 4$; $a_2 = 6$; $b_1 = 6$; $b_2 = 9$; $c_1 = 16$ and $c_2 = 24$

∴ We can say that there are infinite solution or (x, y) has infinite values.

3. (d) Let the cost of each chocolate, each biscuit and each cake be ₹x, ₹y and ₹z respectively.

$3x + 4y + 5z = 34 \longrightarrow$ (i)

$6x + 8y = 38 \Rightarrow 3x + 4y = 19 \rightarrow$ (ii)

$\therefore$ eqⁿ(i) − eqⁿ(ii) ; We get $\rightarrow 5z = 15 \Rightarrow \boxed{z=3}$

---

4. (d) By option test $\rightarrow$

(i) $\frac{2}{3} \Rightarrow \frac{2+1}{3+1} \Rightarrow \frac{3}{4} \Rightarrow \frac{1}{2}$

(ii) Now, $\frac{2}{3} \Rightarrow \frac{2+2}{3+7} \Rightarrow \frac{2}{5}$

$\therefore$ option (d) ► Satisfying the given conditions.

The the Required answer is $\boxed{\frac{2}{3}}$

---

5. (d) By option test $\rightarrow$

(i) In 84 $\rightarrow (8+4) = 3(8-4)$

(ii) Now $\rightarrow (84-48) = 36$

$\therefore$ option (d) satisfying the given Conditions.

Then the required answer is $\boxed{84}$

---

6 (a) Let the time taken by Rakesh is 't' hours.

$\therefore$ Rajesh takes $(t+5)$ hours.

According to the question: $t - \frac{(t+5)}{2} = \frac{15}{2}$

$\Rightarrow \frac{2t-(t+5)}{2} = \frac{15}{2}$

$\Rightarrow t - 5 = 15$

$\Rightarrow t = 20$

Speed of Rajesh :

$\frac{500}{(20+5)} = \boxed{20 \text{ km/hr}}$

[I][A] NUMBER SYSTEM (ANSWERS) (19)

7. (d) $4x + 5y = 32 \longrightarrow$ (i)

$12x + 15y = 2K \longrightarrow$ (ii)

Now; $eq^n (i) \times \frac{3}{2} \rightarrow 6x + 7.5y = 48$

and $eq^n (ii) \times \frac{1}{2} \rightarrow 6x + 7.5y = K$

$\therefore \boxed{K = 48}$

8. (c) Let the numbers are x and y.

$\therefore x + y = 200 \longrightarrow$ (i)

$x^2 - y^2 = 8000 \longrightarrow$ (ii)

Now; $(x+y)(x-y) = 8000 \Rightarrow (x-y) = 40 \rightarrow$ (iii)

$\therefore$ Solving $eq^n (i)$ & $eq^n (iii)$; We get $x = 120$ & $y = 80$

$\therefore$ Required answer is $= \boxed{120}$

9. (c) By option; We can Write $\rightarrow$

(i) $8(6+4) - 16 = 64$

(ii) $22(6-4) + 20 = 64$

$\therefore$ option (c); satisfying the given conditions.

Then, the required answer is $\boxed{64}$

10. (b) Le the two parts be x and y.

$\therefore x + y = 80 \rightarrow$ (i)

$\frac{1}{x} + \frac{1}{y} = \frac{4}{75} \Rightarrow \frac{x+y}{xy} = \frac{4}{75} \rightarrow$ (ii)

By optiontest; putting the respective values $\boxed{30 \text{ and } 50}$ in eq^n(ii); We get $\rightarrow \frac{30+50}{30 \times 50} \Rightarrow \frac{4}{75}$

$\therefore$ optine (b) Satisfying the given conditions.

11. (d) Let the amount of money that Ajay has be ₹ x and that with Vijay be ₹ y.

$\therefore (x - 30) = y \Rightarrow x - y = 30 \longrightarrow$ (i)

and; $(x + 20) = (y - 20) + 70$ $\longrightarrow$ (ii)

$\Rightarrow x - y = 30$

Since; the two equations are the same, the value of x or y cannot be uniquely determined.

$\therefore$ Required answer is $\boxed{(d)}$.

12. (a)

(10 paise coins) (10×70) p ₹ 7

(25 paise coins) (25×70) p ₹ 17.5

10

7.5 : 3

5 : 2

$\therefore$ Required answer is $\left(70 \times \frac{2}{7}\right) = \boxed{20}$

13. (d) Let the fixed expense be ₹ x and the variable expense be ₹ y per head.

$x + 20y = (20 \times 650) \Rightarrow x + 20y = 13000 \longrightarrow$ (i)

and; $x + 25y = (25 \times 600) \Rightarrow x + 25y = 15000 \rightarrow$ (ii)

From eq^n (i) & eq^n (ii); we get $x = 5000$ and $y = 400$

Let the required number of occupants be n.

$\therefore 5000 + 400n = 500n$

$\Rightarrow 100n = 5000$

$\Rightarrow \boxed{n = 50}$

14. (c) By option test; We can write the followings →

(i) $603 - 306 = 297$
(ii) $(3+0) = (6-3)$
(iii) $6 = (2\times 3)$

$\boxed{603}$; satisfying the all conditions.
∴ The Required ans is (c)

15. (a) Let the cost of each pen, ruler and refill be x, y and z respectively.

$$\therefore\ 3x + 4y + 5z = 75 \rightarrow \text{(i)}$$
$$6x + 7y + 10z = 138 \rightarrow \text{(ii)}$$

[eqn(i) × 2] − eqn(ii) → We get → $y = 12$.

$$\therefore\ 3x + y + 5z \Rightarrow (3x + 4y + 5z) - 3y$$
$$= \{75 - (3\times 12)\} = \boxed{39}$$

16. (a) Let the price per kg of Oranges; Mangoes, Bananas and Grapes be ₹R; ₹M; ₹B and ₹G respectively.

$$\therefore\ 5R + 2M = 310 \rightarrow \text{(i)}$$
$$3M + 3.5B = 230 \rightarrow \text{(ii)}$$
$$1.5B + 5G = 610 \rightarrow \text{(iii)}\ +$$

$$\text{(i)} + \text{(ii)} + \text{(iii)} \Rightarrow 5R + 5B + 5G = 700$$

$$\therefore\ 2(5R + 5B + 5G) = (700 \times 2)$$

$$\therefore\ 10R + 10B + 10G = \boxed{₹\ 1400}$$

17. (a) Let the number be x.

$\therefore$ According to the question:- $\left(\frac{7x}{3} - \frac{3x}{7}\right) = 1680$

$\Rightarrow \frac{40}{21}x = 1680 \Rightarrow x = (42 \times 21) = \boxed{882}$

18. (b) Let the number of pencils with Q is $3x$.

$\therefore$ The number of pencils with P is $5x$.

According to the question: $(5x - 3x) = 18$

$\Rightarrow 2x = 18$

$\times 4 \quad 8x = \boxed{72} \quad \times 4$

19 (b) $2x + ky = 1 + 2y \Rightarrow 2x + y(k-2) = 1 \rightarrow$ (i)

and; $kx + 12y = 3 \longrightarrow$ (ii)

From the equations to have infinite number of solution: $\frac{2}{k} = \frac{k-2}{12} = \frac{1}{3}$

$\Rightarrow \frac{2}{k} = \frac{1}{3} \Rightarrow \boxed{k = 6}$

20 (b) Let the sum of ages today be 'x'.

$\therefore$ Sum 18 years ago $= (x - 36)$

$\therefore (x - 36) = \frac{x}{2} \Rightarrow x = 72$

Ratio of present ages $= 2:1$

$\therefore A = \left(72 \times \frac{2}{3}\right) = \boxed{48 \text{ years}}$

21. (b) Two Original tickets = $2x$

Two extra tickets = $2(x+50)$

Total amount = $(4x+100)$

Total money they spent = (4×60) = ₹ 240

∴ According to the question: $(4x+100) = 240$

$\Rightarrow x = 35$

∴ Actual price of each ticket be ₹ x = $\boxed{₹ 35}$

22. (c) $\frac{bx-ay}{b} = \frac{cy-bz}{c} = \frac{az-cx}{a} = K$

$\therefore c(bx-ay) + a(cy-bz) + b(az-cx) = K(ab+bc+ca)$

$\Rightarrow K(ab+bc+ca) = 0 \Rightarrow \boxed{(ab+bc+ca) = 0}$

23. (d) Let x is be the length of the longer piece.

$\therefore x + \frac{4}{7}x = 77 \Rightarrow x = 49$

$\therefore \left(\frac{3}{14} \text{ of } 49\right) = \boxed{10.5}$

24. (a) By optiontest; We can write →

(i) $\{4(1+6) - 12\} = 16$

(ii) $\{2(6-1) + 6\} = 16$

$\therefore$ option (a); satisfying Both of the Conditions.

Then, the required answer is $\boxed{16}$.

---

25. (b) Let the Volumes of Water in X and Y [redacted] be a litres and b litres respectively.

$\therefore \; a = b + 400 \longrightarrow$ (i)

Now; $(a + 200) = 2(b + 200) \longrightarrow$ (ii)

putting the value of 'a' from eqn (i) in eqn (ii);

We get the solution.

$\therefore \; (b + 400 + 200) = 2b + 400$

$b = 200$

$\Rightarrow \; a = 600$

Then then required answer is $\boxed{600 \text{ litres}}$

(E) ALGEBRA (Answers) (20)

1. (b) $x = \sqrt{\dfrac{5+2\sqrt{6}}{5-2\sqrt{6}}}$

$\Rightarrow x = \sqrt{\dfrac{(5+2\sqrt{6})^2}{(5+2\sqrt{6})(5-2\sqrt{6})}} \Rightarrow \sqrt{(5+2\sqrt{6})^2} \Rightarrow 5+2\sqrt{6}$

$\Rightarrow (x-5) = 2\sqrt{6}$

$\Rightarrow (x-5)^2 = (2\sqrt{6})^2$

$\Rightarrow (x-5)^2 = 24$

$\Rightarrow (x-5)^2$

$\Rightarrow x^2 - 10x + 25 = 24$

$\Rightarrow x^2 - 10x = -1$

$\Rightarrow x(x-10) = -1$

$\Rightarrow \boxed{x^2(x-10) = 1}$

2. (d) $x = \left[\dfrac{(1\cdot331)^{-1} + (1\cdot331)^{-2} + \cdots + (1\cdot331)^{-6}}{(1\cdot331)^{-2} + (1\cdot331)^{-3} + \cdots + (1\cdot331)^{-7}}\right]^{\frac{1}{3}} \div 1\cdot1$

$\Rightarrow x = \left[\dfrac{\{(1\cdot331)^{-1} + (1\cdot331)^{-2} + \cdots + (1\cdot331)^{-6}\}}{(1\cdot331)^{-1}\{(1\cdot331)^{-1} + (1\cdot331)^{-2} + \cdots + (1\cdot331)^{-6}\}}\right]^{\frac{1}{3}} \div 1\cdot1$

$\Rightarrow x = (1\cdot331)^{\frac{1}{3}} \div 1\cdot1$

$\Rightarrow x = \{(1\cdot1)^{3\times\frac{1}{3}} \div 1\cdot1\} \Rightarrow \boxed{1}$

3. (b) Now; $a+b = \dfrac{2+\sqrt{3}}{2-\sqrt{3}} + \dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

$\therefore a+b = \dfrac{(2+\sqrt{3})^2 + (2-\sqrt{3})^2}{1} \Rightarrow (a+b) = 14$

Now; $ab = 1$

$a^2 + ab + b^2 \Rightarrow (a^2 + 2ab + b^2) - ab \Rightarrow (a+b)^2 - ab$

Now; $(a^2 + ab + b^2) = \{(14)^2 - 1\} \Rightarrow \boxed{195}$

4 (b) $a\sqrt{7} + b = \dfrac{\sqrt{7}-2}{\sqrt{7}+2}$

$\Rightarrow a\sqrt{7} + b = \dfrac{(\sqrt{7}-2)^2}{7-4} \Rightarrow \dfrac{7+4-4\sqrt{7}}{3} \Rightarrow \dfrac{11}{3} - \dfrac{4\sqrt{7}}{3}$

$\Rightarrow a\sqrt{7} + b = \dfrac{11}{3} - \dfrac{4\sqrt{7}}{3}$

$\therefore$ clearly; $\boxed{a = -\dfrac{4}{3}}$

5. (a) $f(x) = 2x^3 + ax^2 + 3x - 5$

$g(x) = x^3 + x^2 - 2x + a$

By the Remainder Theorem:

$f(2) = [(2 \times 2^3) + (a \times 2^3) + (3 \times 2) - 5] \Rightarrow 17 + 4a$

Again; $g(2) = [2^3 + 2^2 - (2 \times 2) + a] \Rightarrow 8 + a$

$\therefore \quad 17 + 4a = 8 + a \Rightarrow \boxed{a = -3}$

6 (b) $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

$\Rightarrow f(1) = (1 - 2 + 3 - a + b) \Rightarrow (2 - a + b) \left[\because \begin{matrix} x-1=0 \\ x=1 \end{matrix}\right]$

$f(-1) = (1 + 2 + 3 + a + b) \Rightarrow (6 + a + b) \left[\because \begin{matrix} x+1=0 \\ x=-1 \end{matrix}\right]$

$\therefore \quad 2 - a + b = 5$ [redacted] $\Rightarrow (b - a) = 3 \longrightarrow$ (i)

and; $6 + a + b = 19 \Rightarrow (a + b) = 13 \longrightarrow$ (ii)

By Solving eq^n (i) & eq^n (ii); We get $\rightarrow$

$\boxed{a = 5 \; ; \; b = 8}$

7 (c) $\frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\frac{1}{z^2+x^2-y^2}$

Now; $x+y+z=0$

$\therefore$ let put $x=1$; $y=1$ and $z=-2$

$\therefore x+y+z=(1+1-2)=0$

then, $\frac{1}{(1)^2+(1)^2-(-2)^2}+\frac{1}{(1)^2+(-2)^2-(1)^2}+\frac{1}{(-2)^2+(1)^2-(1)^2}$

$=\left(-\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)\Rightarrow\boxed{0}$

8 (c) $a+b+c=2s \Rightarrow$ ~~$a+b=(s-a)+(s-b)$~~

$\Rightarrow c=(s-a)+(s-b)$

$(s-a)^3+(s-b)^3+3(s-a)(s-b)c$

$=(s-a)^3+(s-b)^3+3(s-a)(s-b)[(s-a)+(s-b)]$

$=[(s-a)+(s-b)]^3 \Rightarrow \boxed{c^3}$ $\left(\because c=(s-a)+(s-b)\right)$

9 (c) $x^2+4y^2+z^2+3=2x+4y+2z$

$\Rightarrow (x^2-2x+1)+(4y^2-4y+1)+(z^2-2z+1)=0$

$\Rightarrow (x-1)^2+(2y-1)^2+(z-1)^2=0$

Now; $x-1=0$	$2y-1=0$	$z-1=0$
$x=1$	$y=\frac{1}{2}$	$z=1$

Then; $(x+y+z)=\left(1+\frac{1}{2}+1\right)\Rightarrow\boxed{2\frac{1}{2}}$

10. (a) $(x+y+z)=0$

put $x=1$; $y=-1$ and $z=0$

$x^2(y+z)+y^2(z+x)+z^2(x+y)+3xyz$

$=(-1+1+0+0) \Rightarrow \boxed{0}$

11. (c) [redacted] $3\left(\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}\right)$

$=3\left(\frac{a}{a^2+a}+\frac{b}{b^2+b}+\frac{c}{c^2+c}\right)$

$=3\left(\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}\right)$

$=3\left(\frac{a+b+c}{a+b+c}\right)=\boxed{3}$

12. (d) $x+\frac{1}{x}=5$

$\Rightarrow x^2+1=5x$

$=\frac{x^4+\frac{1}{x^2}}{x^2-3x+1}$

$\frac{x^4+\frac{1}{x^2}}{(x^2+1)-3x}$

$\frac{x^4+\frac{1}{x^2}}{5x-3x}$

$\frac{1}{2x}\left(x^4+\frac{1}{x^2}\right)$

$=\frac{1}{2}\left(x^3+\frac{1}{x^3}\right)$

$=\frac{1}{2}\left[\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)\right]$

$=\frac{1}{2}\left[(5)^3-3.5\right]$

$=\boxed{55}$

13. (b) $\frac{3-5x}{x}+\frac{3-5y}{y}+\frac{3-5z}{z}=0$

$\Rightarrow \frac{3}{x}-5+\frac{3}{y}-5+\frac{3}{z}-5=0$

$\Rightarrow 3\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=15$

$\Rightarrow \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\boxed{5}$

14 (c) $x^5 + \frac{1}{x^5}$

$= \left(x^3+\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right) - \left(x+\frac{1}{x}\right)$ $\left\{\because \left(x+\frac{1}{x}\right) = 3\right\}$

$= \left[\left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)\right]\left[\left(x+\frac{1}{x}\right)^2 - 2\right] - \left(x+\frac{1}{x}\right)$

$= \left[3^3 - 3.3\right]\left[3^2-2\right] - 3 \Rightarrow \{(18 \times 7) - 3\} \Rightarrow \boxed{123}$

15. (a) $\frac{a^3+b^3+c^3-3abc}{a+b+c} = (a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow \frac{a^3+b^3+c^3-3abc}{(a+b+c)} = \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$\Rightarrow \frac{a^3+b^3+c^3-3abc}{a+b+c} = \frac{1}{2}\left[3^2+5^2+1^2\right] \Rightarrow \boxed{17.5}$

16. (a) $(ax+by)^2 + (bx-ay)^2 = 40$

$\Rightarrow a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2 = 40$

$\Rightarrow (x^2+y^2)(a^2+b^2) = 40$

$\Rightarrow (a^2+b^2) = \frac{40}{(x^2+y^2)}$

$= \left(\frac{40}{4}\right)$

$= \boxed{10}$

17. (c) $a+\frac{1}{a} = 1 \Rightarrow a^2+1-a = 0 \Rightarrow (a+1)(a^2-a+1) = 0$

$\therefore a^3+1 = 0 \Rightarrow a^3 = \boxed{-1}$

18. (a) $a + \frac{1}{(a+2)} = 0 \Rightarrow (a+2) + \frac{1}{(a+2)} = 2$

Now; $(a+2)^3 + \frac{1}{(a+2)^3} \Rightarrow \left[(a+2)+\frac{1}{(a+2)}\right]^3 - 3\left[(a+2)+\frac{1}{(a+2)}\right]$

$\therefore (a+2)^3 + \frac{1}{(a+2)^3} \Rightarrow \{(2)^3 - 3.2\} \Rightarrow \boxed{2}$

19 (c) $a = \sqrt{2}+1$; $b = \sqrt{2}-1$; then $ab = 1$

Let put $a = 1$; $b = 1$

$\therefore \frac{1}{a+1} + \frac{1}{b+1} \Rightarrow \left(\frac{1}{2}+\frac{1}{2}\right) = \boxed{1}$

20. (d) $a^4 + a^2b^2 + b^4 = 8$

$\Rightarrow (a^2)^2 + 2a^2b^2 + (b^2)^2 - a^2b^2 = 8$

$\Rightarrow (a^2+b^2)^2 - (ab)^2 = 8$

$\Rightarrow (a^2+b^2+ab)(a^2+b^2-ab) = 8$

$\Rightarrow (a^2+b^2+ab) = 4$

$\therefore (a^2+b^2-ab) = 2$

Now $\cancel{(a^2+b^2)} + ab = 4$

$\underline{\cancel{(a^2+b^2)} - ab = 2}$
$\quad - \qquad\; + \quad\; -$

$2ab = 2$

$\boxed{ab = 1}$

21. (b) $x = 3 + 2\sqrt{2}$
$\Rightarrow x = 3 + 2\cdot\sqrt{2}\cdot 1$
$\Rightarrow x = (\sqrt{2})^2 + (1)^2 + 2\cdot\sqrt{2}\cdot 1$
$\Rightarrow x = (\sqrt{2}+1)^2$
$\Rightarrow \sqrt{x} = \sqrt{2}+1$
$\Rightarrow \frac{1}{\sqrt{x}} = \sqrt{2}-1$
$\sqrt{x} - \frac{1}{\sqrt{x}}$
$= \cancel{\sqrt{2}}+1 - \cancel{\sqrt{2}}+1$
$= \boxed{2}$

22. (a) $x^2 + y^2 - z^2 + 2xy$
$= (x^2 + y^2 + 2xy) - z^2$
$= (x+y)^2 - (z)^2$
$= (x+y+z)(x+y-z)$
putting the Values of x, y and z
then Weget $(x+y+z) = 0$
$\therefore$ Required answer is $\rightarrow \boxed{0}$

23. (a) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$
$\Rightarrow \left(x^2 + \frac{1}{x^2} - 2\right) + \left(y^2 + \frac{1}{y^2} - 2\right) = 0$
$\Rightarrow \left(x^2 + \frac{1}{x^2} - 2\cdot x\cdot\frac{1}{x}\right) + \left(y^2 + \frac{1}{y^2} - 2\cdot y\cdot\frac{1}{y}\right) = 0$
$\Rightarrow \left(x - \frac{1}{x}\right)^2 + \left(y - \frac{1}{y}\right)^2 = 0$
$\therefore x - \frac{1}{x} = 0$ | $y - \frac{1}{y} = 0$
$\Rightarrow x^2 = 1$ | $\Rightarrow y^2 = 1$
$\therefore x^2 + y^2 = \boxed{2}$

24. (a) $\frac{x-y}{} = \frac{x+y}{7} = \frac{xy}{4} = K$
$\therefore x - y = K$ and $x + y = 7K$
$\therefore x = 4K$ and $y = 3K$
$xy = 4K$
$\Rightarrow 12K^2 = 4K$
$\Rightarrow K = \frac{1}{3}$
$xy = 4K$
$(4 \times \frac{1}{3})$
$= \boxed{\frac{4}{3}}$

25. (C) $ax^2 + bx + c = a(x-p)^2$
$\Rightarrow ax^2 + bx + c = a(x^2 + p^2 - 2xp)$
$\Rightarrow ax^2 + bx + c = ax^2 + ap^2 - 2axp$
Compairing the Corresponding coefficients
$b = -2ap$;
$\Rightarrow b^2 = 4a^2p^2$
$\Rightarrow p^2 = \frac{b^2}{4a^2}$
$c = ap^2$
$\Rightarrow p^2 = \frac{c}{a}$
$\therefore \frac{b^2}{4a^2} = \frac{c}{a}$
$\Rightarrow \boxed{b^2 = 4ac}$