1(d)

Unit place digit = unit place digit of $(3\times5\times1\times7)$
$= 105$
So, unit place digit = 5.

2(c)

We know that $x^n - a^n$ is divisible by $(x + a)$.

So, $(15^{100} - 1^{100})$ is divisible by $(15 + 1) = 16$

$(15^{100} - 1)$ is divisible by 16.

So, if 15^{100} is divided by 16 then the remainder = 1.

3(d)

The number should be divisible by 4 (2×2), 11, 12 (4×3) 33 (11×3) and 132 (12×11).
So, if the number is divisible by 4, 11 and 3 then it is divisible by the rest.
So, by using all the options given above, 999900 is the greatest 6-digit number exactly divisible by 4, 11, 12, 33 and 132.

4(c)

587 and 157 are prime numbers.
$667 = 23 \times 29$.
$561 = 3 \times 11 \times 17$.
$329 = 7 \times 47$.
So, 561 has the maximum number of divisors.

5(d)

Since 45 is completely divisible by 15, then the part of the dividend that is divisible by 45 will also, be divisible by 15. So, when the same number is divided by 15, the remainder $= \frac{18}{15}$, i.e. 3.

6(b)

$$1 + 2 + 3 \ldots + 27 = \frac{27(27+1)}{2} = 378$$

$$1 + 2 + 3 + \ldots + 26 = \frac{26(26+1)}{2} = 351 .$$

So, $1 + 2 + 3 \ldots + 27 + 26 + 25 + \ldots + 3 + 2 + 1$
$= 378 + 351 = 729$.

7(d)

Divisibility rule of 3 is: if the sum of digits of a number is divisible by 3, then the number is also divisible by 3.
Now, $8 + 7 + 3 + 9 + 2 + 1 = 30$.
Hence, the largest possible value of $x = 9$.

8(c)

Since $203 = 29 \times 7$,
The reminder $= 90 = (29 \times 3) + 3$.
So, remainder $= 3$.

9(c)

The difference between squares of consecutive numbers is equal to the sum of numbers, i.e. $769 + 768 = 1{,}537$.

10(b)

997 is the greatest 3-digit prime number.

11(B)

1 is neither a prime nor a composite number.

12(B)

Even digits up to $138 = \frac{138}{2} = 69$.

The sum of even digits up to
$69 = n\,(n + 1) = 69 \times 70 = 4{,}830$.

13(B)

Let $x = 71$.
Then $(x + 13)\ (x - 13) = (71 + 13)\ (71 - 13)$
$= (84)\,(58) = 4{,}872$.
When 4,872 is divided by 71, the remainder is 44.

14(C)

$7^3 \times 6^1 \times 10^2 \times 4^4 \times 5^6$
$= 7^3 \times 2^1 \times 3^1 \times 5^2 \times 2^2 \times 2^8 \times 5^6 = 2^{11} \times 3^1 \times 5^8 \times 7^3$.
Prime factors $= 11 + 1 + 8 + 3 = 23$.

15(d)

x	y	sum
1	48	49
2	24	26
3	16	19
4	12	16
6	8	14

Therefore, clearly that 14 is minimum possible value.

16(a)

(1 + 2 + … + 96) (1 + 2 + … + 40)

$$\left[\frac{96\times97}{2}\right]-\left[\frac{40\times41}{2}\right]$$

4,656–82 = 3,836

According to the question

$$\frac{3{,}836}{17}=225\frac{11}{17}$$

So, the remainder is 11.

17(d)

a = 8P and b = 7Q.

if P = 19, then a = 152.

if Q = 22, then b = 154.

So, Q – P = 22 – 19 = 3.

18(d)

Let the other number be = x.

$$\text{then } (x)\left(c^6\ d^5\right)=\frac{c^7}{d}$$

$$x=\frac{c^7}{d}\times\frac{1}{c^6d^5}=\frac{c}{d^6}$$

19(b)

Odd numbers between 40 and 50 are 41, 43, 45, 47 and 49.

Sum = 41 + 43 + 45 + 47 + 49= 225.

20(c)

913 = 11 × 83

So, 83 is the largest prime number that exactly divides 913.

21(c)

$80000000 - 800 = 79999200.$

22(a)

1 is neither a composite number nor a prime number.
$91 = 13 \times 7$, $213 = 71 \times 3$, $133 = 19 \times 7$ and $119 = 17 \times 7$.

23(a)

$$18 \times 125 \times 63 \times 4 \times 17 = (2 \times 3^2) \times (5^3) \times (7 \times 3^2) \times (2^2) \times (17)$$
$$= 2^3 \times 3^4 \times 5^3 \times 7 \times 17$$

Zeroes are formed by 2×5

So, the number of the zeroes = 3

24(a)

$$1^3 + 2^3 + 3^3 + \ldots + 50^3 = 1{,}625{,}625$$
$$7 + 56 + 189 + \ldots + 8{,}75{,}000 = 7$$

$$(1 + 8 + 27 + \ldots + 1{,}25{,}000)$$
$$= 7\left(1^3 + 2^3 + 3^3 + \ldots + 50^3\right)$$
$$= 7 \times 16{,}25{,}625$$
$$= 11{,}379{,}375$$

25(a)

$$240 = 2^4 \times 3^1 \times 5^1$$

Factors including 1 and 240 is
$$= (4 + 1)(1 + 1)(1 + 1)$$
$$= 5 \times 2 \times 2 = 20$$

Factors excluding 1 and 240 = $20 - 2 = 18$.

26(a)

The HCF of 6.3, 1.33 and 56.7 = HCF of $\left(\frac{63}{10}, \frac{133}{100}, \frac{567}{10}\right)$.

$$\text{HCF of}\left(\frac{63}{10}, \frac{133}{100}, \frac{567}{10}\right) = \frac{\text{HCF of }(63, 133, 567)}{\text{LCM of }(10, 100, 10)}$$

$63 = 7 \times 3^2$

$133 = 7 \times 19$

$567 = 7 \times 3^4$

So, HCF of 63, 133 and 567 is 7.

$$= \frac{7}{100} = 0.07$$

27(b)

The required number = HCF of [(103 − 5), (170 − 16) and (275 − 37)]
= HCF of 98, 154 and 238 = 14

28(a)

The required number
= HCF of [(755 − 159) and (1,947 − 755)]
= HCF of (596, 1,192)
= 596

29(c)

If HCF of two numbers = 6,
Let the numbers be $6x$ and $6y$.
$6x + 6y = 42$
$x + y = 7$
Possible pairs are (1, 6), (2, 5), (3, 4), i.e., three pairs are possible.

30(b)

The required number
= HCF of [(50 − 8), (90 − 6) and (110 − 5)]
= HCF of (42, 84 and 105)
Hence, the required number is 21.

31(d)

$5 - 2 = 7 - 4 = 9 - 6 = 11 - 8 = 3$
So, the required number
= LCM of (5, 7, 9 and 11) + 3
= 3,465 + 3
= 3,468

32(a)

In order to find the minimum number of containers we need to find the maximum capacity of the container.
The maximum capacity = HCF of (66, 132 and 154) = 22.

$$\text{So, the number of containers} = \frac{66}{22} + \frac{132}{22} + \frac{154}{22} = 3 + 6 + 7 = 16.$$

33(d)

Time taken by A = $\frac{2,400}{15}$ = 160 seconds

Time taken by B = $\frac{2,400}{20}$ = 120 seconds

Time taken by C = $\frac{2,400}{30}$ = 80 seconds

The LCM of 160, 120 and 80 = 480 seconds = 8 minutes.
Hence, they will be together at the same place after 8 minutes.

34(c)

The LCM is always divisible by HCF but 960 is not divided by 420, Hence, 420 cannot be the HCF.

35(b)

LCM of 4, 6 and 12 = 12
Now the greatest four digit number which exactly divides 4, 6 and 12 is 9,996
So, the required number = 9,996 + 3 = 9,999

36(b)

The required number is HCF of [(888 – 416), (1,537 – 888) and (2,245 – 1,537)]
= HCF of (472, 649 and 708)
= 59

37(c)

To find when all the four bells toll together, we have to find their LCM. LCM of (18, 45, 36, 144)
= 720 seconds
= 12 minutes

38(d)

The greatest possible length = HCF of (33, 55 and 77) = 11 m

39(b)

LCM of 45, 60 and 36 = 180

Hence, they will meet after 180 seconds = 3 minutes.

40(d)

$8 - 5 = 15 - 12 = 54 - 51 = 3$

So, the required number

$= \text{LCM of } (8, 15 \text{ and } 54) - 3$

$= 1{,}080 - 3$

$= 1{,}077$

41(a)

LCM of (30, 45 and 60) = 180

Thus, lights will change simultaneously after $\frac{180}{60} = 3$ minutes.

So, at 11:30 a.m. they will change simultaneously.

42(d)

The greatest measure of the container = HCF of (184, 168 and 104) = 8

43(c)

$(15 - 10) = (21 - 16) = (35 - 30)$

$= (70 - 65) = 5$

So, required number

$= \text{LCM of } (15, 21, 35, 70) - 5$

$= 210 - 5 = 205$

44(a)

LCM of (15, 30, 18, 45) = 90

So, the required number = 90 + 11 = 101.

45(a)

Required number is HCF of [(597 − 7) and (650 − 2)]

$= \text{HCF of } (592 \text{ and } 648)$

$= 8$

46(d)

Required number

$= \text{LCM of } (9, 27 \text{ and } 36) + 5$

$= 108 + 5$

$= 113$

47(d)

LCM of 3, 4, 5, 6 and 10 = 60
So, the required number = $60 \times 15 = 900$.

48(d)

The required number = HCF of [(111 − 62), (202 − 111) and (321 − 202)]
= HCF of (49, 91 and 119)
= 7

49(b)

LCM of 1,000 natural numbers = N
Now, up to 1,010, all numbers have their factors upto 1,000 except 1,009.
So, LCM of the first 1,010 natural numbers = $N \times 1{,}009$.

50(c)

HCF of $2^4 \times 3^5 \times 6^3 \times 7^2$,
$2^5 \times 3^2 \times 6^1 \times 7^5$ and
$2^3 \times 3^4 \times 6^2 \times 7^1 \times 9^2$ is
$= 2^3 \times 3^2 \times 6^1 \times 7^1$